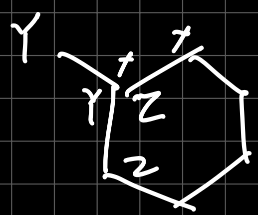
Physics 211C: Solid State Physics

Instructor: Prof. Tarun Grover

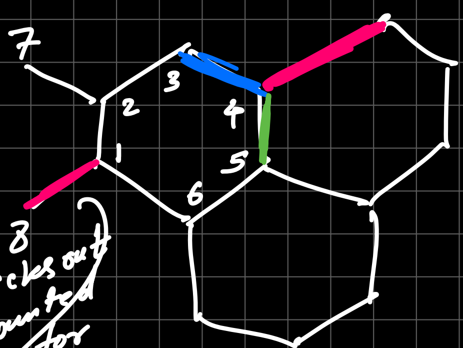
Lecture 17

Topic: Kitaev's honeycomb model

$Li_2 Ir_1, O_3$, α-$RuCl_3$, ... $\Rightarrow$ Kitaev materials



* Model:



whatever sticks out gets accounted for

$$J_x \sum_{\substack{\langle ij\rangle \\ \in \text{red}}} X_i X_j + J_y \sum_{\substack{\langle ij\rangle \\ \in \text{blue}}} Y_i Y_j + J_z \sum_{\substack{\langle ij\rangle \\ \in \text{black}}} Z_i Z_j$$

$$W_p = X_1 Y_2 Z_3 X_4 Y_5 Z_6$$

$$[W_p, H] = 0$$

$$X_1 X_8, \quad Z_1 Z_2, \quad Y_1 Y_6$$

$$(-1)\times(-1) \qquad (-1)\times(-1)$$

$$W_p^2 = +1 \Rightarrow W_p = \pm 1$$

Hilbert space size in a given W_p sector of fixed eigenvalues

$$= \frac{2^{N_{sites}}}{2^{N_{plaquettes}}} \qquad N_{plaquettes} = \frac{3N_s}{6} = \frac{N_s}{2}$$

$$= 2^{N_s/2} = (\sqrt{2})^{N_s}$$

turns out low en. theory is given

by Majorana fermion

$\dim(\mathcal{H}) \overset{\text{"}}{=} \sqrt{2}$ in some sense

$$C = \frac{\eta_1 + i\eta_2}{2}$$

$\eta_1^+ = \eta_1$

$\eta_2^+ = \eta_2$

$\eta_1^2 = \eta_2^2 \neq 0 = \text{const.} = \frac{1}{2}$

$\eta_1 \eta_2 + \eta_2 \eta_1 = 0$

$\{c, c^+\} = \mathbb{1}$

$\Rightarrow cc^+ + c^+c = \mathbb{1}$

$\{\eta_1 + i\eta_2, \eta_1 - i\eta_2\}$

$= \frac{1}{4}[2\eta_1^2 + 2\eta_2^2] = \mathbb{1}$

$\Rightarrow \boxed{\eta_1^2 + \eta_2^2 = 2}$

Solution

define

b^x, b^y, b^z, c on each site $\rightarrow$ 4 majorana fermions

$(b^x)^2, (\ )^2 \ldots = \mathbb{1}$

$X = i b^x c$

$Y = i b^y c$

$Z = i b^z c$

Actual dim = 2 (Hilbert space of spin $\frac{1}{2}$)

dim now: $(\sqrt{2})^4 = 4$

const. $XY = iZ$

$\Rightarrow \boxed{XYZ = i} \Rightarrow \boxed{b^x b^y b^z c = \mathbb{1}}$ $\rightarrow$ squares to 1 so ev is ± 1

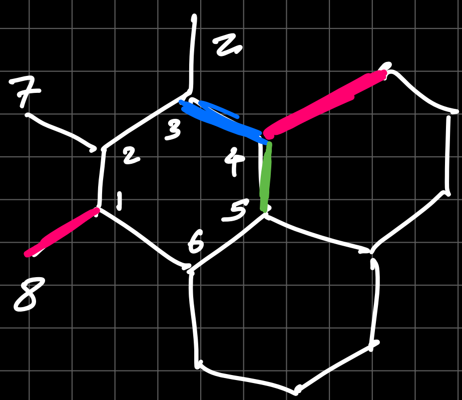
but if we pick one ev $\rightarrow$ makes Hilbert space is $\frac{1}{2}$ed.

$$\mathcal{H} = \sum J_x\, x_i x_j + J_y\, y_i y_j + J_z\, z_i z_j$$

$$\mathcal{H} = \sum_{\langle ij \rangle} i\, J_{ij}\, u_{ij}\, c_i c_j$$

where $= i\, b_i^{\alpha} b_j^{\alpha}$

$\alpha = (x, y, z)$ depending on the bond orientatn



idea:- U_{ij} commutes with the Hamiltonian.

$$u_{12} = i\, b_i^z b_j^z$$

$z_1 z_2 \qquad x_2 x_3$

$$[z_1 z_2, u_{12}] \overset{?}{=}$$

$$z_1 z_2 = i^2\, b_1^z c_1\, b_2^z c_2 = b_1^z b_2^z c_1 c_2 \qquad u = i\, b_1^z b_2^z$$

clearly commutes

$$x_2 x_3 = b_1^x b_2^x c_1 c_2$$

$$u_{12} = b_1^z b_2^z \qquad [x_2 x_3, u_{12}] = 0$$

$$P_{physical}\, [u_{ij}, \mathcal{H}]\, P_{physical} = 0$$

$u_{ij} \approx \mathbb{Z}_2$ gauge field

$\mathcal{H} =$ free fermion within a given sector of u_{ij}.

Lieb's theorem:- g.s. lies in the zero flux sector.

set $u_{ij} = 1$

$$\prod_{\hexagon} u_{ij} = W_p$$

Toric code:- $-\sum_{\square}\underbrace{\prod_{\square} Z}_{\sim W_p} - \sum_{+}\prod_{+} X$

$b_i^x b_i^y b_j^z c = \mathbb{1} \Rightarrow$ very much like Gauss's law

$b_i^x b_j^y b_i^z c_i$: anticommutes with both u_{ij} & $c_i c_j$.

$$|g.s.\rangle = \prod_i \left[\frac{1+b_i^x b_i^y b_i^z c_i}{2}\right] |\text{Free fermion GS}\rangle$$

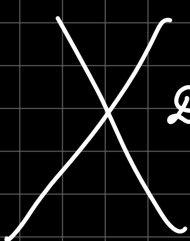
g.s. of $\underbrace{\sum_{\langle ij\rangle} J_{ij}\, c_i c_j}$

$\Downarrow$

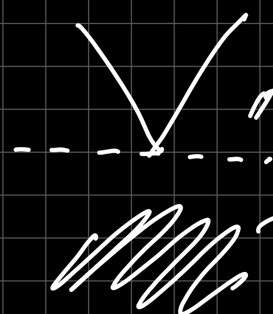
gives a similar "dirac" like structure

here we set $J_x, J_y, J_z = \mathbb{1}$

$i\sum J_{ij}\, c_i c_j \longrightarrow$ no antiparticle. $\therefore$ only $\frac{1}{2}$ of a dirac fermion.



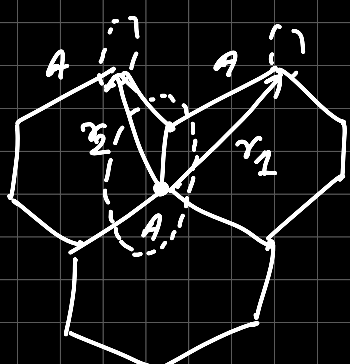
Dirac cone



Majorana cone

$\rightarrow$ anti-particles disappears.

Band structure:-



$$c = \frac{1}{\sqrt{N_s}}\sum_k c_k e^{ikr}$$

$$\begin{bmatrix} c_A^\dagger & c_B^\dagger \end{bmatrix} \begin{bmatrix} 0 & i(1+e^{ikr_1}+e^{ikr_2}) \\ \underbrace{i(1+e^{-ikr_1}+e^{-ikr_2})} & 0 \end{bmatrix} \begin{pmatrix} c_A \\ c_B \end{pmatrix}$$

$\Downarrow$

$c_k^\dagger = c_{-k}$ not $c_k^\dagger = c_k$

$if(q)$, $-if^*(q)$ $\longrightarrow$ nodes & expand around nodes to get

Dirac fermion.

$$k = \left(\pm \frac{4\pi}{3\sqrt{3}}, 0 \right)$$

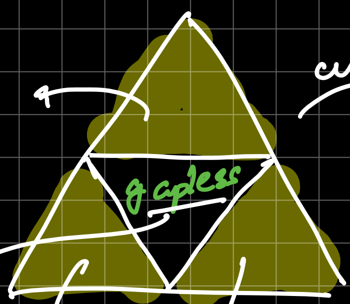
Taylor expand around node:-

$$\begin{bmatrix} c_A^\dagger & c_B^\dagger \end{bmatrix} \begin{bmatrix} 0 & q_x + i q_y \\ q_x - i q_y & 0 \end{bmatrix} \begin{bmatrix} c_A \\ c_B \end{bmatrix}$$

Low en. theory:-

gapless neutral fermions coupled to a $\mathbb{Z}_2$ gauge fields.

gapped



cut in $\to J_x + J_y + J_z = 1$

gapped

gapped $\to \mathbb{Z}_2$ coupled to gapped Majorana

$\simeq$ Toric code

not truly topological

$$1, e, m, \underbrace{em}_{\text{fermion}} = c \text{ (i.e. gapless)}$$

ising CFT?